Math 199 Merit **Worksheet 11** **November 3, 2016**

Sections 10.2, 10.3

You may need to use the following angel sum and difference identities for Problem 2.

i. $\cos(\theta \pm \beta) = cos(\theta)cos(\beta) \mp \sin(\theta)\sin(\beta)$

ii. $\sin(\theta \pm \beta) = \sin(\theta)cos(\beta) \pm cos(\theta)sin(\beta)$

Integral of even and odd functions (you may need for Problem 2, 3,4.)

i. If f is an odd function which is integrable in the interval $[-L, L]$, then $\int\limits_{-L}^{L} f(x)dx = 0$.

ii. If f is an even function which is integrable in the interval $[-L, L]$, then

$$\int\limits_{-L}^{L} f(x)dx = 2\int\limits_{0}^{L} f(x)dx.$$

1. Periodic functions

Determine whether the given functions is periodic. If, so find it fundamental period.

(a) $\sin(\frac{m\pi x}{L})$ **(b)** $\cos(\frac{m\pi x}{L})$ **(c)** $\tan(3\pi x)$ **(d)** $x^2 + 3x$

(e) $f(x) = \begin{cases} (-1)^n, & 2n-1 \le x < 2n \\ 1, & 2n \le x < 2n+1; \end{cases} \qquad n = 0, \pm 1, \pm 2, \ldots$

2. Orthogonality of sine and cosine functions

(a) Show that $\int_{-\pi}^{\pi} \cos(mx)\cos(nx)dx = \int_{-\pi}^{\pi} \sin(mx)\sin(nx)dx = 0$ for integers $m \neq n$.

(b) Show that $\int_{-\pi}^{\pi} \cos(mx)\sin(nx)dx = 0$ and

$$\int_{-\pi}^{\pi} \cos^2(nx)dx = \int_{-\pi}^{\pi} \sin^2(nx)dx = \pi$$

3. Fourier series

Assume that there is a Fourier series converging to the function f defined by

$$f(x) = \begin{cases} -x, & -3 \le x < 0 \\ x, & 0 \le x < 3; \end{cases}$$

$$f(x+6) = f(x).$$

Determine the coefficients in this Fourier series.

4. Initial Value Problems

Find the formal solution of the initial value problem

$$y'' + \omega^2 y = f(t),\ y(0) = 1,\ y'(0) = 0,$$

where f is periodic with period 2 and

$$f(t) = \begin{cases} 1 - t, & 0 \leq t < 1; \\ -1 + t, & 1 \leq t < 2. \end{cases}$$

What can be done by using Fourier series ?